Prepared by
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Rensselaer Polytechnic Institute

The Diffraction Tube
and Catalog No. 2639A Power Supply

A permanently evacuated, multiple target electron-diffraction, cathode-ray tube with which students measure the wavelength of electrons to discover for themselves the dual character of matter.

The tube was developed with the cooperation of the General Electric Company by Harry F. Meiners and Stanley A. Williams for use in the Sophomore Physics Laboratories of Rensselaer Polytechnic Institute. It is produced under special arrangement by The General Electric Company for exclusive distribution by The Welch Scientific Company.

WELCH SCIENTIFIC COMPANY
7500 NORTH LINCOLN AVENUE, SKOKIE, ILLINOIS
The Permanently evacuated Multiple-target Electron-diffraction Cathode-ray Tube

Two different target materials are provided to double usefulness of the tube.

TRANSMISSION PATTERNS

2639A POWER SUPPLY. For No. 2639 Electron Diffraction Tube. (With tube shown in place.)
INTRODUCTION

In 1924 de Broglie\textsuperscript{1} predicted that the wavelength of matter waves could be found by using the same relationship that held for light, namely

\[ \lambda = \frac{\hbar}{p}, \tag{1} \]

where \( \lambda \) is the wavelength of a light wave, \( \hbar \) is Planck's constant and \( p \) is the momentum of the photons. De Broglie proposed that matter, as well as light, has a dual character behaving in some circumstances like particles and in others like waves. He suggested that a particle of matter having a momentum \( p \) would have an associated wavelength \( \lambda \).

The first experimental evidence of the existence of matter waves was obtained by Davisson and Germer\textsuperscript{2} in 1927. They "reflected" slow electrons from a single crystal of nickel and applied de Broglie's relationship (See Eq. 4). The wavelength of the electrons was determined and compared with that calculated from Bragg's\textsuperscript{3} expression (See Eq. 5). Excellent agreement was obtained.

The wavelength of a beam of electrons, as indicated above, is given by

\[ \lambda = \frac{p}{\hbar} = \frac{b}{mv}, \]

where \( (mv) \) is the momentum of a moving electron. The wavelength is therefore inversely proportional to the velocity \( v \) of the electrons. This velocity can be obtained directly from the accelerating potential \( V \) in the vacuum tube. Since the kinetic energy of the electrons is given by

\[ \frac{1}{2}mv^2 = eV \tag{2} \]

the wavelength

\[ \lambda = \frac{\hbar}{\sqrt{2meV}} \tag{3} \]

where \( m \) is the mass and \( e \) is the charge of the electron. When the values of \( \hbar \), \( m \), and \( e \) are substituted, this becomes

\[ \lambda = \frac{\sqrt{150}}{V} \tag{4} \]

if \( \lambda \) is expressed in angstroms and \( V \) in volts. Since our apparatus is operated in general below 10 KV, electron energies are non-relativistic and no relativistic correction factor is needed.

The results of Thomson's\textsuperscript{4} experiments with fast electrons supplied additional evidence about the behavior of electrons. Thomson analyzed photographically diffraction patterns produced by electron beams passing through thin films of gold, aluminum and other materials. From measurements of the size of the electron diffraction rings on a fluorescent screen, the wavelength of the electrons was found and again agreed with that predicted by de Broglie's equation. Other experimental work has shown that all particles have dual properties.

1. L. de Broglie, \textit{Phil. Mag.} 47, 446 (1924)
This experiment uses Thomson's method of transmitting electrons through a thin film of randomly oriented crystals to investigate a ring diffraction pattern with the aid of Bragg's law,

\[ 2d \sin \theta = n \lambda \]  

(5)

In this equation, \( d \) is the separation between lattice planes, \( \lambda \) is the wavelength of the electrons, \( \theta \) is the ordinary angle between the incident beam and the reflecting plane, which is the same as the angle between the reflected beam and the reflecting plane, and \( n \) is the order of the reflection. The angle between the direct and diffracted beams is \( 2 \theta \), as shown in Fig. 1.

1. \( 2\theta = \frac{r}{D} \)
2. \( 2a^* \sin \theta = (h^2 + k^2 + l^2)^{\frac{1}{2}} \)
3. \( \chi = \frac{d \theta}{D} \)

Fig. 1. Ring diffraction pattern produced by the constructive interference of the electron waves diffracted from the various families of planes within the randomly oriented crystals in the thin film aluminum target.

Fig. 2. The face-centered cubic lattice of aluminum. Two possible reflecting planes are shown. The separation between the (100) planes is called the lattice constant \( d_{100} = a^* \).

The separation \( d \) between reflection planes for a face-centered cubic structure (FCC), such as aluminum, in terms of the Miller indices (HKL) is

\[ d = \frac{a^*}{\sqrt{H^2 + K^2 + L^2}} \]  

(6)

where \( a^* \) is the length of the edge of the unit cell (Fig. 2). When the Miller indices are multiplied by the order of the reflection \( n \), any order \( n \) of Bragg reflection from planes (HKL) is considered to be first order Bragg reflection from planes \( \{hkl\} \).

Therefore,

\[ d = \frac{na^*}{\sqrt{b^2 + k^2 + l^2}} \]  

(7)

where \( nH = h, nK = k \) and \( rL = l \). Substituting into Eq. 5, we obtain
\[
2\frac{\lambda^*}{\sin \theta} = \lambda^* = \frac{\lambda^*}{\sqrt{h^2 + k^2 + l^2}}. \tag{8}
\]

When the angle \( \theta \) in Eq. 8 is small, the \( \sin \theta \) can be replaced with \( \theta = \frac{r}{2D} \).
\[
\theta = \frac{r}{2D}. \tag{9}
\]

and Eq. 8 becomes
\[
\lambda^* = \frac{d^* r}{D} \frac{1}{\sqrt{h^2 + k^2 + l^2}}. \tag{10}
\]

where \( D \) is the distance from target to screen and \( r \) is the radii of rings. \( \lambda^* \) is known from x-ray measurements. \( D \) and \( r \) are obtained by direct measurements.

When comparing the wavelength calculated from de Broglie's relationship with that calculated from Bragg's expression, it is helpful to designate the two values as \( \lambda \) and \( \lambda^* \), respectively, so that they can be tabulated without confusion. \( \lambda \) and \( \lambda^* \) represent the wavelength of the same electrons.

The observed diffraction pattern consisting of rings of various radii is produced by the constructive interference of the electron waves diffracted from the various families of planes within the randomly oriented crystals in the thin film target. The intensity of a reflection in the diffraction pattern is proportional to the square of the corresponding structure factor, i.e.,
\[
I(\h k \ell) \propto |F(\h k \ell)|^2.
\]

For a face-centered cubic structure, 6
\[
F(\h k \ell) \sim 1 + e^{i\pi(\h+k)} + e^{i\pi(\k+l)} + e^{i\pi(\l+h)}. \tag{11}
\]
The structure factor actually takes into consideration the coordinates and differences in scattering power of the individual atoms, the Miller indices \( h k \ell \), and the addition of sine waves of different amplitude and phase but of the same wavelength. When squared, the structure factor vanishes unless \( h, k, \ell \) are all odd or even, in which case \( F = 0 \). (See Table I for allowed FCC reflections for aluminum.) Therefore rings will occur for which \( h, k, \ell \) are all odd or even.

<table>
<thead>
<tr>
<th>( h k \ell )</th>
<th>( \sqrt{h^2 + k^2 + l^2} )</th>
<th>( \sqrt{\frac{h^2 + k^2 + l^2}{3}} )</th>
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</table>

Using methods first applied by Ewald\textsuperscript{7} and von Laue,\textsuperscript{8} spot patterns from hexagonal pyrolytic graphite will be investigated. Measurements are made directly from the tube screen (See Fig. 3).

\begin{equation}
\begin{aligned}
\theta &= 30^\circ \\
1. & \quad \lambda = \frac{1}{\sqrt{V}} \\
2. & \quad d = \frac{\lambda D}{r} \\
3. & \quad a = \frac{\lambda D}{r \cos \theta}
\end{aligned}
\end{equation}

Fig. 3. Analysis of hexagonal spot diffraction pattern. The second equation is derived from Bragg's relationship. \(d\) is the separation between planes, \(D\) is the distance from target to screen. The distance \(r\) from the central spot to each of the spots on the hexagon is measured. \(a\) is the calculated lattice constant which is compared to the known \(a^*\) found by x-ray measurements.

To obtain the separation \(d\) between planes,\textsuperscript{9} first calculate the wavelength \(\lambda\) of the electrons from de Broglie's relationship (Eq. 4) and substitute in Bragg's relationship, \(\lambda\) for \(\lambda^*\). For small angles the latter becomes

\[d = \frac{\lambda D}{r}.\]  \hspace{1cm} (12)

Once \(d\) is determined, the lattice constant \(a\) for hexagonal pyrolytic graphite can be calculated and compared with the known constant \(a^*\) obtained from x-ray measurements. Conversely, given \(d^*\), the wavelength \(\lambda^*\) can be calculated and compared with \(\lambda\) computed from the voltage.

**OPERATING PROCEDURE**

Warning! Do not touch connecting wires at the rear of the power supply or tube. High voltage is dangerous. If for any reason the tube must be moved or wires disconnected, call your instructor. The tube is evacuated to a pressure of \(10^{-8}\) mm Hg and should be handled with caution to avoid implosion.

1. A microammeter should be connected to the external meter jack on the power supply and used at all times to measure target currents. The range will depend upon which application is being used, as indicated below. (The required meter is assumed to be available in any physics laboratory and is therefore not included with the power supply.)

7. P. P. Ewald, Z. Krist. 56, 120 (1921)
2. **Using tube for a laboratory experiment.** Target current is usually 5 to 10 microamperes and should not exceed 10 microamperes. Gun filament and target are designed to have a long life when operated under normal laboratory conditions with subdued illumination. **POST CURRENT RECOMMENDATIONS ON TOP OF POWER SUPPLY FOR STUDENT REFERENCE.**

3. **Using for demonstrations.** A target current of 50 to 100 microamperes is suggested. Continued operation at high current may be expected to shorten the life of the tube.

4. **Using for closed-circuit television.** Target currents of 10 to 25 microamperes should be adequate but will depend upon the type of TV equipment being used and will require some testing to get optimum conditions. The phosphor on the tube screen was selected so that both ring and spot diffraction patterns could be picked up with maximum effect using relatively low target currents. Tube and camera should be shielded somewhat from room illumination, with black paper as a hood for example.

5. **Before turning on the power supply, turn the intensity (bias) and voltage controls to "off".**

   Turn the A.C. line switch on. Allow a few minutes for the power supply and the tube filament to warm up. Then turn the high voltage control to the desired value.

6. **To avoid burning the phosphor screen, turn the intensity (or bias) control on slowly. Do not exceed current ratings. Do not at any time increase the intensity beyond the point where the power supply overloads and the high voltage meter dips. Watch the central beam spot for possible burning of the screen due to prolonged high intensity at one position. This will not occur if the centering controls are used to move the beam slowly while searching for diffraction patterns. If suggested target current ratings are followed, screen will not be burned when beam is focused on a particular target.**

7. **Always adjust focus control until spot size on screen is as small as possible for each selected high voltage reading.**

8. **Turn the intensity and voltage controls to "off" position before switching off A.C. line voltage.**

**SUMMARY**

This tube will last a long time if it is used properly. Instructors should become thoroughly familiar with its characteristics before allowing students to take data. Always work with the minimum current needed to give good patterns for the purpose desired. Keep the spot moving slowly while searching for a good target. When the beam is focused on a target, intensity can be increased without screen damage. Correct the voltage after using the centering and focusing controls since there is some interaction between controls. A much more expensive supply would be needed to eliminate all interaction.
Experiment 1. Analysis of the Electron Diffraction Pattern of Polycrystalline Aluminum

\[ \lambda = 1.732 \times 10^{-7} \, \text{Å} \]

1. Look up the value of the lattice constant \( a^* \) which is known from x-ray measurements.

2. Using a series of voltages to be suggested by the instructor, measure the radii of all observed circular rings at each voltage. So that the effects of ring distortion may be minimized, obtain an average of four measurements of the radius of each ring by measuring across four different diameters. Caution! Check the voltage continuously while making measurements or correcting focus. A.C. line voltage may vary and affect meter readings. Record all your measurements, including the voltages.

3. Bring a small magnet near the face of the tube. The diffraction pattern is deflected with little distortion demonstrating that the pattern is formed by electrons and not electromagnetic radiation. Use the magnet carefully. If the electron beam is deflected greatly, internal arcing might result.

4. Apply Bragg's relationship and calculate the wavelength \( \lambda^* \) of the electrons for all permitted reflections (See Table I) at each voltage selected in paragraph 2. For the same voltages compute from de Broglie's equation the wavelength \( \lambda \). Record all reflections and the corresponding values of \( \lambda^* \) and \( \lambda \).

Since some allowed reflections cannot be observed because of their low intensity, check carefully for an obvious lack of agreement between \( \lambda^* \) and \( \lambda \). For example, if the choice of the Miller indices for a particular measurement of the radius \( r \) of a ring pattern results in an unusual spread between \( \lambda^* \) and \( \lambda \), then try other combinations of the indices until good agreement between \( \lambda^* \) and \( \lambda \) is obtained.

5. Discuss your results. Explain why the ring diameters increase with decrease in voltage.

\[ a^* = 0.4769 \, \text{Å} \]

\[ D = 7.00 \, \text{cm} \]

SUGGESTED DATA TABLE FOR EXPERIMENT 1

| Accelerating Potential (Volts) | Reflection Plane | \( \left( k^2 + l^2 + l^2 \right) \) | \( r \) (cm) | \( \lambda (Å) \) | \( \lambda^* (Å) \) | \%
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<td>311</td>
<td>3.316</td>
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</table>

Note: Repeat table for each voltage selected. Include under \( r \) the four measurements of the radius of each ring and the mean \( r \).

\[ 1.23 \, \text{cm} \rightarrow 1.736 \times 10^{-7} \, \text{Å} \]

\[ 1.54 \, \text{cm} \rightarrow 1.738 \times 10^{-7} \, \text{Å} \]
Experiment 2. Calculation of the Lattice Constant of Aluminum

1. Determine the number of atoms associated with a unit cell for a face-centered cubic lattice. Use the molecular weight of aluminum $M$, its density $\rho$, Avogadro's number $N_0$, the number of atoms in the unit cell, and calculate the lattice constant $a^+$.

2. Substitute for $\lambda^*$ in Bragg's relationship, each $\lambda$ obtained from de Broglie's relation for the various voltages used in Experiment 1. Calculate the lattice constant $a$ for the allowed combinations of reflections. Record all values of $a$ and the mean $\bar{a}$. Also include the corresponding voltages.

3. Compare the known value of the lattice constant $a^*$, obtained from x-ray measurements, with the mean $\bar{a}$, found from Bragg's relationship, and $a^+$, calculated above in paragraph 1. Discuss.

**SUGGESTED DATA TABLE FOR EXPERIMENT 2**

<table>
<thead>
<tr>
<th>Accelerating Potential (Volts)</th>
<th>Reflection Plane $(h^2 + k^2 + l^2)^{1/2}$</th>
<th>$r$ (cm)</th>
<th>$\lambda$ (Å)</th>
<th>$a$ (Å)</th>
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<td>$\overline{r}$ =</td>
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<td>$\overline{r}$ =</td>
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**Note.** Repeat table for each voltage selected.
Experiment 3. Analysis of the Spot Electron Diffraction Patterns of Pyrolytic Graphite

1. Derive from Bragg’s relationship an expression which can be used to calculate the wavelength \( \lambda^* \) of the electrons. The equation should include the known lattice constant \( a \) of hexagonal pyrolytic graphite which is 2.456 \( \text{Å} \) obtained from x-ray measurements.

2. For a series of voltages to be recommended by the instructor, measure the distance \( r \) from the central beam spot to each of the spots in the hexagonal diffraction pattern. Caution! Check the voltage continuously while making measurements or correcting focus. Record each measurement \( r \) and the mean \( \bar{r} \) for the corresponding voltages.

3. Use the mean \( \bar{r} \) of the spot measurements to calculate the wavelength \( \lambda^* \) of the electrons. Also compute the wavelength \( \lambda \) by applying de Broglie’s relationship for each voltage. Record \( \lambda^* \) and \( \lambda \).

4. Does the agreement between \( \lambda^* \) and \( \lambda \) fall within the limits of accuracy of the apparatus? Discuss.

**SUGGESTED DATA TABLE FOR EXPERIMENT 3**

<table>
<thead>
<tr>
<th>Accelerating Potential (Volts)</th>
<th>( r ) (cm)</th>
<th>( d ) (Å)</th>
<th>( \lambda^* ) (Å)</th>
<th>( \lambda ) (Å)</th>
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<tbody>
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<tr>
<td>( \bar{r} )</td>
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</table>

Note. Repeat table for each voltage selected.

Experiment 4. Calculation of the Lattice Constant of Pyrolytic Graphite

1. Substitute for the wavelength \( \lambda^* \) in Bragg’s relationship \( \lambda \) obtained from de Broglie’s equation. Compute the lattice constant \( a \). Do this for each voltage selected in Experiment 3 or for a series of voltages recommended by your instructor. If voltages other than those used in Experiment 3 are selected, measure the distance \( r \) from the central beam spot to each of the spots in the hexagonal diffraction pattern. Be sure to check the voltage continuously while making measurements or correcting the focus. Record each measurement \( r \) and the mean \( \bar{r} \) for the corresponding voltages. Also record the calculated lattice constant \( a \) for each voltage.
2. Compare the value of the lattice constant \( a^* \) known from x-ray measurements with the mean \( a \) of your calculated \( a \) values. Discuss.

**SUGGESTED DATA TABLE FOR EXPERIMENT 4**

<table>
<thead>
<tr>
<th>Accelerating Potential (Volts)</th>
<th>( r ) (cm)</th>
<th>( d(A) )</th>
<th>( \lambda(A) )</th>
<th>( a(A) )</th>
<th>( % )</th>
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<td>( \bar{r} )</td>
<td></td>
<td></td>
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</tbody>
</table>

Note. Repeat table for each voltage selected.

**QUESTIONS**

1. Prove that \( \lambda(A) = \frac{\sqrt{150}}{V} \).

2. Would the above equation be valid for a 10 Mev electron beam? If not, why not? Derive an equation which would apply.

3. How is the wavelength related to the electron's kinetic energy?

4. Assume that the electron beam is replaced with a beam of positively charged particles. Could you carry out the same analysis if the particles were positrons? What would be the result if the particles were protons?

5. Plot a graph of \( \lambda^2 \) vs \( \frac{1}{V} \), where \( \lambda \) is in angstroms and \( V \) is in volts. Use data obtained from de Broglie's relationship. What is the significance of the slope?

6. Derive an expression for the separation \( d \) between planes for a simple cubic lattice.

7. Why is it necessary for the inner wall of the tube to be covered almost completely with a conducting coating, i.e., graphite?

8. Why does the diffraction pattern consist of concentric rings? Would a single crystal produce the same pattern? Explain.

9. Why are the target, screen, and conducting coating on the inner wall of the tube connected to ground?

10. Assuming that Helmholtz coils are placed on each side of the tube, derive an expression based on pattern deflection which could be used to calculate the speed of the electrons. Also derive in terms of the accelerating voltage an alternate
equation for the speed $v$ of the electrons. Could the ratio of $e/m$ for the electrons be found in this manner? Explain.

11. Explain why an understanding of matter waves is important in the study of wave mechanics.

REFERENCES


